Simulation of the discharge curve by short-time discharge based on the power of internal resistance

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Abstract

The actual discharge curve of the power source depends upon the voltage, the capacity, the load, and upon the condition of the power source at the time of use. A reliable power supply from the chemical power source can be achieved by simulation of its discharge curve. The methodology of simulation of discharge curves by set: time t (s) versus discharge voltage, U_d (V), from a short-time discharge (<1% of total discharge time) through the constant load at the constant temperature have been developed. The two basic equations were defined: (i) discharge current, U_d/R_l (A), $U_{d_n}/R_l = a_n t_n^{b_n}$ and, (ii) potential drop, P_d (V), by introducing estimating voltage, U (V) instead of the open-circuit of potential, U_o (V): $U - U_{d_n} = c_n t_n^{d_n}$. From these equations, the equation for the power of internal resistance P_i (V A) can be derived: $[U_{d_n}(U - U_{d_n})]/R_l = a_n c_n t_n^{(b_n + d_n)}$. The equation for the power of internal resistance was used to simulate the discharge curve. Alkaline manganese cells LR 20-VARTA, loads: 10 Ω and 3.33 Ω, were used for the demonstration of method. The mathematical calculations were conducted on an IBM personal computer using Symphony software. The method was developed to test the quality of the produced cells and batteries, reliable exploitation of all systems of chemical power sources, and optimization of the battery charging. The method is a fast, nondischarging, nondestructive and repetitive technique.

Introduction

The prediction of cell/battery performance and reliability can be achieved by the determination of discharge time by the short-time discharge [1] or preferably, by simulation of the discharge curve. Method of simulating, a discharge curve by pair set: time, t (s), versus discharge voltage, U_d (V), by short-time discharge (<1% of the total discharge time) through the constant load at the constant temperature will be developed.

The two basic equations are defined:

(i) discharge current, I_{d_n} (A)

$$\frac{U_{d_n}}{R_l} = a_n t_n^{b_n} \tag{1}$$

and,

(ii) potential drop, P_d, by introducing the estimating voltage, U (V) instead of the open-circuit voltage, U_o (V)

$$U - U_{d_n} = c_n t_n^{d_n} \tag{2}$$

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where a_n, c_n are coefficients, b_n, d_n the exponents from the regression analysis, $n=1$, 2, 3...n the ordinal number of the pair set: time, t_n (s) versus discharge voltage, U_{d_n} (V); R_l (Ω) is the load.

From eqns. (1) and (2) the two following equations are obtained:

(i) reciprocal internal resistance, $(1/R_i)$ (Ω^{-1}) [1]:

$$\frac{U_{d_n}}{R_l(U-U_{d_n})}=\frac{a_n}{c_n}t_n^{(b_n-d_n)} \tag{3}$$

and,

(ii) power of internal resistance, P_i (V A):

$$\frac{U-U_{d_n}}{R_l}U_{d_n}=a_n c_n t_n^{(b_n+d_n)} \tag{4}$$

The power of internal resistance of a cell/battery is the complement value to the discharge power by constant resistance [2].

The method is intended to test the quality of produced cells/batteries, reliable exploitation of power sources and optimization of battery charging.

The method is a fast, nondischarging, nondestructive and repetitive technique.

From the simulated, as well as from the real discharge curve, the operating characteristic of the cell/battery can be derived.

Also, there is the possibility to examine the internal resistance of the cell/battery, including its components [3].

Theoretical background

The simulation of the discharge curve is based on eqn. (4), the power of the internal resistance.

For the power curve fit, the regression exponents and the regression coefficients are:

$$b_n=\frac{\sum_1^n\left[\ln\frac{U_{d_n}}{R_l}\ln(t_n)\right]-\frac{1}{n}\sum_1^n\ln\frac{U_{d_n}}{R_l}\sum_1^n\ln(t_n)}{\sum_1^n[\ln(t_n)]^2-\frac{1}{n}\left[\sum_1^n\ln(t_n)\right]^2} \tag{5}$$

$$a_n=\exp\left[\frac{1}{n}\sum_1^n\ln\frac{U_{d_n}}{R_l}-b_n\frac{1}{n}\sum_1^n\ln(t_n)\right] \tag{6}$$

$$d_n=\frac{\sum_1^n[\ln(U-U_{d_n})\ln(t_n)]-\frac{1}{n}\sum_1^n\ln(U-U_{d_n})\sum_1^n\ln(t_n)}{\sum_1^n[\ln(t_n)]^2-\frac{1}{n}\left[\sum_1^n\ln(t_n)\right]^2} \tag{7}$$

$$c_n=\exp\left[\frac{1}{n}\sum_1^n\ln(U-U_{d_n})-b_n\frac{1}{n}\sum_1^n\ln(t_n)\right] \tag{8}$$

From eqn. (4) the discharge time, t (s), can be expressed as:

$$t_n = \left[\frac{U_{d_n}(U-U_{d_n})}{R_l a_n c_n}\right]^{1/(b_n+d_n)} \tag{9}$$

Equation (4) may be differentiated as:

$$\frac{dt}{d[U_d(U-U_d)R_l^{-1}]} = \frac{1}{(b+d)ac}\frac{t}{t^{(b+d)}} \tag{10}$$

$$\frac{dt}{dU_d} = \frac{1}{(b+d)ac}\frac{t}{t^{(b+d)}}\frac{(U-2U_d)}{R_l} \tag{11}$$

$$\frac{dt}{dU_d} = \frac{1}{b+d}\left[\frac{U_d(U-U_d)}{R_l ac}\right]^{1/(b+d)}\frac{U-2U_d}{(U-U_d)U_d} \tag{12}$$

The estimating voltage U (V) may be expressed from eqn. (4) and named the simulating voltage, U_s (V):

$$U_{s_n} = U_{d_n} + R_l \frac{a_n c_n t_n^{(b_n+d_n)}}{U_{d_n}} \tag{13}$$

According to eqns. (7) and (8) the two sets $(c_u, d_u)_n = f(U, \ldots)_n$ and $(c_s, d_s)_n = f(U_s, \ldots)_n$ may be calculated.

The simulating voltage, U_s (V), is the function of the discharge voltage and discharge time:

$$\frac{dU_s}{dU_d} = 1 - R_l \frac{act^{(b+d)}}{U_d^2} \tag{14}$$

$$\frac{dU_s}{dt} = R_l \frac{(b+d)act^{(b+d)}}{tU_d} \tag{15}$$

Experimental

The analysis was conducted on experimental data obtained previously [1, 4]. The cells, LR 20-VARTA, were discharged continuously through constant loads (10 Ω and 3.33 Ω) at room temperature, to the cutoff voltage, $U_{d_n} = U_o/2$.

The mercury relay was used [5, 6] to close circuit over the period 0.1 and 0.3 s.

An HP 3054 DL computer was used to determine and record pairs: discharge time, t_{e_n} (s) versus discharge voltage, U_{d_n} (V).

The following pairs were recorded:

(i) the short-time discharge, $0 < t_e$ (s) $\leqslant 24$, $\Delta t_e = 1$ s, $1 \leqslant k \leqslant 20$.

(ii) the period to the end of the discharge, $U_d = U_o/2$

$-\Delta t_e = 360$ s, $R_l = 3.3$ Ω, $21 \leqslant n \simeq 300$, and

$-\Delta t_e = 2160$ s, $R_l = 10.0$ Ω, $21 \leqslant n \simeq 150$.

The number of the recorded pairs was too large for computing by PC (540 KB RAM) and were reduced to $n = 20 + 60$. The values t_{e_k} (s) were selected by step: ΔU_d (V) $\equiv$ constant.

In this way, for one cell/battery, discharged through the constant load at the constant temperature, the unique pair set: t_e (s) versus U_d (V) was recorded.

TABLE 1

Discharge voltage: absolute errors over a short-time discharge

Error	Sample 1	Sample 2	Sample 3
Maximum	3.8922×10^{-4}	3.6302×10^{-4}	4.6618×10^{-4}
Average	1.7779×10^{-4}	1.7815×10^{-4}	2.4385×10^{-4}
Minimum	5.2042×10^{-18}	5.2042×10^{-18}	7.3726×10^{-18}
	Sample 4	Sample 5	Sample 6
Maximum	1.3794×10^{-3}	7.6312×10^{-4}	1.0888×10^{-3}
Average	6.5608×10^{-4}	2.5646×10^{-4}	3.2331×10^{-4}
Minimum	3.9031×10^{-18}	3.6863×10^{-18}	0

The acquisition of experimental data lacked accuracy and precision due to the small resolution.

The quadratic eqn. (4) was solved:

$$U_{\mathrm{ds},n} = \frac{U_o + \sqrt{U_o^2 + 4R_l(act^{(b+d)})_n}}{2}, \quad 1 \ll n \leqslant 20 \tag{16}$$

where $b=f(U_d, R_l, t_e)$, $a=f(U_d, R_l, t_e, b)$,

$d=f(U_o, U_d, t_e)$, and $c=f(U_o, U_d, t_e, d)$

using $t_n = t_{e_n}$ (s) and $U_{\mathrm{d_e},n}$ (V) and giving the simulated discharge voltage, $U_{\mathrm{d_s},n}$ (V).

Experimental data show significant errors. Over the short-time discharge the maximum, average and minimum absolute errors: $|e|=|U_{\mathrm{d_e}}-U_{\mathrm{d_s}}|$ are given in Table 1. Also, the jump from $t_{e_{k-20}}=24$ s to $t_{e_{n-21}}=2160$ s, with $R_l=10\ \Omega$ and from $t_{e_{k-20}}=24$ s to $t_{e_{n-21}}=360$ s, with $R_l=3.33\ \Omega$, caused difficulties.

In this article the short-time discharge was therefore extended to $1 \leqslant k \leqslant n=21$ i.e., $t_{e_{k-21}}=2160$ s for $R_l=10\ \Omega$ and $t_{e_{k-21}}=360$ s for $R_l=3.33\ \Omega$.

The mathematical calculations were conducted on an IBM PC using Symphony software.

Equations to be of use

The simulation of the discharge curve is based on the simultaneous calculation of both the simulating voltage, U_{s_n} (V) and the simulated discharge time, t_{s_n} (s) at the step n by the values from the step $(n-1)$, i.e.:

$$U_{\mathrm{sx},n} = U_{\mathrm{d}(n-1)} + R_l \frac{(ac_u)_{(x,n-1)} tx_{(x,n-1)}^{(b+d_u)(x,n-1)}}{U_{\mathrm{d}_{n-1}}} + \left[1 - R_l \frac{(ac_u)_{(x,n-1)} tx_{(x,n-1)}^{(b+d_u)(x,n-1)}}{U_{\mathrm{d}(n-1)}^2}\right](U_{\mathrm{d}_n} - U\mathrm{d}_{(n-1)}) \tag{17}$$

and,

$$t_{\mathrm{s}(x,n-1)} = \left[\frac{U_{\mathrm{s}(x,n-1)} - U_{\mathrm{d}(n-1)}}{(ac_s)_{(x,n-1)}R_l} U_{\mathrm{d}(n-1)}\right]^{1/(b+d_s)(x,n-1)} \tag{18}$$

$$t_{s_{x,n}} = t_{s(x,n-1)}\left[1 + \frac{(U_{s(x,n-1)} - 2U_{d(n-1)})(U_{d_n} - U_{d(n-1)})}{(b+d_s)_{(x,n-1)}(U_{s(x,n-1)} - U_{d(n-1)})U_{d(n-1)}}\right] \tag{19}$$

where letter x and subscript x, according to the eqns. (5) to (8) denote whether the coefficients $(a, c_u, c_s)_{e,(k)n}$ and the exponents $(b, d_u, d_s)_{e,(k)n}$ were calculated by the experimental discharge time: $t_{e(k)n}$ (s) or the coefficients $(a, c_u, c_s)_{s,(k)n}$ and the exponents $(b, d_u, d_s)_{s,(k)n}$ were calculated by the simulated discharge time $t_{s_{s,(k)n}}$ (s).

Summary and in the other notation

(i) The simulating voltage, $U_{s_{e,k}}$ (V) and the simulated discharge time, $t_{s_{e,k}}$ (s) were calculated by:

$$U_{s_{e,k}} = f\{a_{(e,k-1)}[b_{(e,k-1)}(R_l, U_{d_k}, t_{e(k-1)})], c_{u(e,k-1)}[d_{u(e,k-1)}(U, U_{d_k}, t_{e(k-1)})]\} \tag{20}$$

$$t_{s_{e,k}} = f\{a_{(e,k-1)}[b_{(e,k-1)}(R_l, U_{d_k}, t_{e(k-1)})], c_{u(e,k-1)}[d_{u(e,k-1)}(U_{s(e,k-1)}, U_{d_k}, t_{e(k-1)})]\} \tag{21}$$

over the interval of a short-time discharge: $3 \leqslant k \leqslant n_{std}$ or over the interval of total discharge period: $3 \leqslant k \leqslant n_{ttl}$.

(ii) The simulating voltage, $U_{s(s,n)}$ (V) and the simulated discharge time, $t_{s(s,n)}$ (s) were calculated by:

$$U_{s_{s,n}} = f\{a_{(s,n-1)}[b_{(s,n-1)}(R_l, U_{d_n}, t_{e(s,n-1)})], c_{u(s,n-1)}[d_{u(s,n-1)}(U, U_{d_n}, t_{e(s,n-1)})]\} \tag{22}$$

$$t_{s_{s,n}} = f\{a_{(s,n-1)}[b_{(s,n-1)}(R_l, U_{d_n}, t_{s(s,n-1)})], c_{s(s,n-1)}[d_{s(s,n-1)}(U_{s(s,n-1)}, U_{d_n}, t_{s(s,n-1)})]\} \tag{23}$$

over the interval from the short-time discharge to the end-of-discharge period $(k_{std}+1) \leqslant n \leqslant n_{ttl}$.

Results

At the moment of the cell/battery test, a primary battery is characterized by its voltage, capacity and the rate capability [2].

The capacities $C_{e(k)n}$ (A s), $C_{s_{e,k}}$ (A s) and $C_{s_{s,(k)n}}$ (A s) were calculated by the corresponding discharge time using the numerical integration method, [7], see Table 2.

In this article, the estimating voltage, U (V) was defined by the following criteria [8]:

$$t_{s_{e,23}} = t_{s_{s,23}} = t_{e23} \pm 0.01t_{e23} \tag{24}$$

and,

$$C_{s_{e,23}} = C_{s_{s,23}} = C_{e23} \pm 0.01C_{e23} \tag{25}$$

The results of the simulation are:

(i) The simulating discharge times

- t_{s1} (s) $= t_{e1}$ (s) and t_{s2} (s) $= t_{e2}$ (s);
- $t_{s_{e,k}}$ (s) over the interval: $3 \leqslant k \leqslant 80$, eqn. (21), and
- $t_{s_{s,n}}$ (s) over the interval: $22 \leqslant n \leqslant 80$, eqn. (23).

TABLE 2

Alkaline manganese LR 20-VARTA
Experimental C_{en} (A s) and calculated $C_{se,n}$ (A s), $C_{ss,n}$ (A s)
Short-period discharge, $n = 1 \ldots 21$
Half of the discharge: $U_d = 3U_o/4$, $n = 40$
Discharge time, $U_d = U_o/2$, $n = 80$

n	C_{en} (A s)	$C_{se,n}$ (%)	$C_{ss,n}$ (%)	C_{en} (A s)	$C_{se,n}$ (%)	$C_{s(s,n)}$ (%)
	Sample 1			Sample 2		
20	3.58	101.43	101.43	3.58	100.26	100.26
21	328.79	104.81	104.81	329.28	99.88	99.88
22	648.58	100.02	100.02	649.64	100.78	100.78
23	693.70	101.72	101.72	965.29	103.46	104.16
40	20969	98.38	129.07	20438	106.53	117.60
80	36825	96.82	96.54	36743	102.01	95.86
	Sample 3			Sample 4		
20	3.59	100.30	100.30	10.41	101.13	101.13
21	329.80	101.88	101.88	158.58	97.82	97.82
22	650.78	99.00	99.00	313.28	100.66	100.66
23	967.05	101.01	101.01	465.88	99.53	100.00
40	21001	101.91	121.00	8151	97.22	105.35
80	36850	102.14	97.99	29981	97.92	98.85
	Sample 5			Sample 6		
20	10.30	101.45	101.45	10.43	100.93	100.93
21	157.20	96.77	96.77	158.99	99.45	99.45
22	310.86	101.07	101.07	314.22	103.40	103.40
23	462.47	99.35	100.90	467.34	100.48	102.07
40	7401	99.82	106.65	8384	99.01	99.06
80	28758	102.25	102.29	28973	105.49	96.03

(ii) The simulating voltages:

- U_{s1} (V) = U (V) and $U_{s2} = U$ (V);
- U_{sn} (V) over the interval: $3 \leqslant n \leqslant 80$, eqn. (13) using the experimental discharge times, t_{en} (s);
- $U_{se,k}$ (V) over the interval: $3 \leqslant k \leqslant 80$, eqn. (20), and
- $U_{ss,n}$ (V) over the interval: $22 \leqslant n \leqslant 80$, eqn. (22).

Figures 1 (sample 1) and 2 (sample 4) show the curve 1, t_e (s), curve 2, $t_{se,k}$ (s), and curve 3, $t_{ss,n}$ (s).

Figures 3 (sample 1) and 4 (sample 4) show the curve 1, U_s (s), curve 2, $U_{se,k}$ (s) and curve 3, $U_{ss,n}$ (s).

The t_s (s) values: $1 \leqslant k \leqslant 21+2$, $n = 40$ (i.e., $U_{d40} = 3U_o/4$) and $n = 80$ (i.e., $U_{d80} = U_o/2$) were tabulated in Tables 3 to 5.

Only the values $t_{ss,n}$ (s) and $C_{ss,n}$ (A s) over the range $(k_{std}+1) = (21+1) \leqslant n \leqslant 80$ belonged to simulation of discharge curve by short-time discharge method.

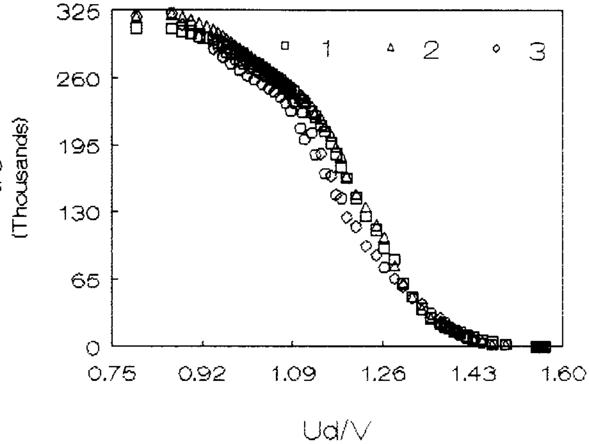

Fig. 1. Discharge curves, LR 20-VARTA, $R_l = 10\ \Omega$. Sample 1: $U_o = 1.5905$ V, $U = 1.5728$ V. Curve 1, experimental, t_e (s), curve 2, simulated, $t_{ss,k}$ (s) eqn. (21), and curve 3, simulated, $t_{ss,n}$ (s) eqn. (23).

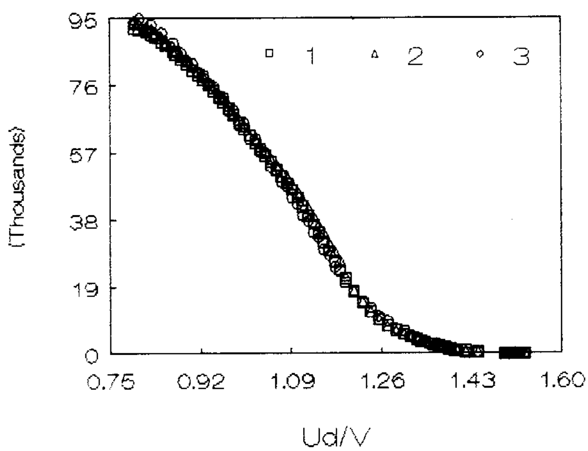

Fig. 2. Discharge curves, LR 20-VARTA, $R_l = 3.33\ \Omega$. Sample 4: $U_o = 1.5901$ V, $U = 1.5518$ V. Curve 1, experimental, t_e (s), curve 2, simulated, $t_{se,k}$ (s), eqn. (21), and curve 3, simulated, $t_{ss,n}$ (s), eqn. (23).

The values $t_{se,k}$ (s) and $C_{se,k}$ (A s) are calculated for the analysis. Over the range $1 \leqslant k \leqslant 21$, the first term in eqn. (19), i.e., eqn. (18), is the experimental value $t_{e(k-1)}$ (s), and then $t_{se,k}$ (s) $= t_{ss,k}$ (s).

The tabulated time values (Tables 3 to 5) and capacity values (Table 2) demonstrate a good accordance between experimental and simulated data. At the beginning and the end of the discharge period, the relative errors are smaller than 6%. At the middle of the discharge interval (i.e., $U_d \cong 3U_o/2$) the errors are significant, more than 29% for sample 1. For the samples 4, 5 and 6, the relative errors are smaller than 6%.

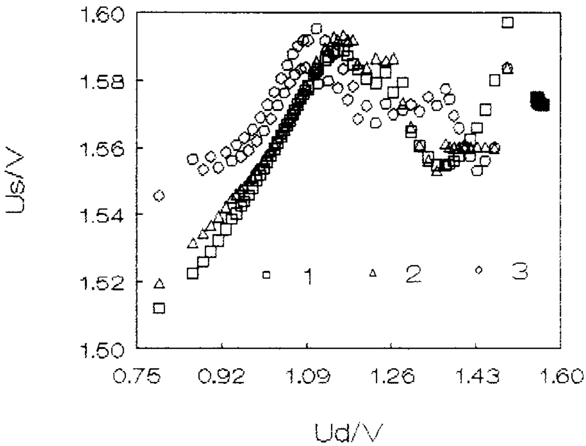

Fig. 3. Simulating voltages, LR 20-VARTA, $R_l = 10$ gU. Sample 1: $U_o = 1.5905$ V, $U = 1.5728$ V. Curve 1, experimental, U_{s_n} (V), eqn. (13), curve 2, simulated, $U_{se,k}$ (V), eqn. (20), and curve 3, simulated, $U_{ss,n}$ (s), eqn. (22).

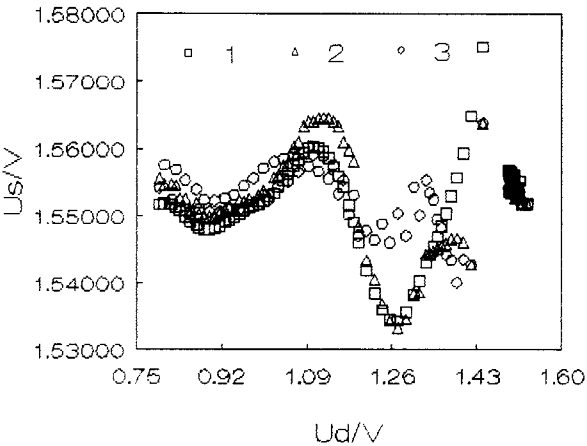

Fig. 4. Simulating voltages, LR 20-VARTA, $R_l = 3.33\ \Omega$. Sample 4: $U_o = 1.5901$ V, $U = 1.5518$ V. Curve 1, experimental, U_{s_n} (V), eqn. (13), curve 2, simulated, $U_{se,k}$ (V), eqn. (20), and curve 3, simulated, $U_{se,n}$ (s), eqn. (22).

Discussion

The implicit relationship between the discharge voltage U_d (V) (as independent variable) and discharge time, t (s) (as dependent variable) is based on the definition of the power of cell/battery internal resistance and expressed as quadratic formula, eqn. (4). The cell/battery internal resistance, R_i (Ω) may be divided [6] into several components: ohmic resistance, R_{ohm}, reaction resistance, R_{react}, and diffusion resistance, R_{diff}. In this way, eqn. (4) becomes:

$$\frac{U_o - U}{R_l} U_d + \frac{U_s - U_d}{R_l} U_d = act^{(b+d)} \tag{26}$$

TABLE 3

Alkaline manganese LR 20-VARTA
Experimental U_d (V) vs. t_e (s) and simulated time: t_{se} (s), t_{ss} (s)
Short-period discharge, $n=1...21$
Half of the discharge: $U_d=3U_o/4$, $n=40$
Discharge time, $U_d=U_o/2$, $n=80$

	Sample 1				Sample 2			
	$R_l=10\ \Omega$, $U=1.5728$ V				$R_l=10\ \Omega$, $U=1.56945$ V			
n	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)
0	1.5905	0	0	0	1.5907	0	0	0
1	1.5669	1	1	1	1.565	1	1	1
2	1.5629	3	3	3	1.5626	3	3	3
3	1.5611	4	4.16	4.16	1.5613	4	4.45	4.45
4	1.5598	5	4.95	4.95	1.5603	5	5.26	5.26
5	1.5588	6	5.80	5.80	1.5595	6	6.10	6.10
6	1.558	7	6.70	6.70	1.5588	7	7.02	7.02
7	1.5573	8	7.66	7.66	1.5583	8	7.78	7.78
8	1.5566	10	8.71	8.71	1.5577	10	8.98	8.98
9	1.5561	11	10.57	10.57	1.5572	11	10.91	10.91
10	1.5555	12	11.74	11.74	1.5568	12	11.77	11.77
11	1.555	13	12.66	12.66	1.5564	13	12.81	12.81
12	1.5546	14	13.56	13.56	1.556	14	13.85	13.85
13	1.5542	16	14.60	14.60	1.5557	16	14.67	14.67
14	1.5538	17	16.66	16.66	1.5553	17	16.96	16.96
15	1.5534	18	17.69	17.69	1.555	18	17.75	17.75
16	1.553	19	18.73	18.73	1.5547	19	18.79	18.79
17	1.5527	20	19.58	19.58	1.5544	20	19.82	19.82
18	1.5524	21	20.60	20.60	1.5541	21	20.85	20.85
19	1.5521	23	21.63	21.63	1.5539	23	21.59	21.59
20	1.5518	24	23.68	23.68	1.5536	24	23.94	23.94
21	1.4932	2160	2060.88	2060.88	1.496	2160	2162.47	2162.47
22	1.4678	4320	4321.88	4321.88	1.4703	4320	4285.80	4285.80
23	1.4487	6481	6371.88	6372	1.451	6481	6260	6217
40	1.1929	162367	164863	124747	1.1930	158047	147486	133052
80	0.7953	308160	319068	319208	0.7954	310000	304792	327621

where U_o (V) and U (V) are the constant and U_s (V) is the second dependent variable. The first term represents R_{ohm} and the second term represents sum of R_{react} and R_{diff}.

Equation (4) (or eqn. (26)) can be solved by several mathematical techniques. Only the tangent approximation was used in this article.

U_s (V) as the second dependent variable may be eliminated from eqns. (18) and (19) using eqn. (17) showing that the simulated discharge times, $t_{ss,n}$ (s) were calculated by the values from the two previous steps, i.e., $(n-2)$ and $(n-1)$.

The $t_{ss,n}$ (s) and $U_{ss,n}$ (V) curves are not smooth because $\Delta U_{d(n-1)}=(U_{d_n}-U_{d(n-1)})$, $3 \leqslant n \leqslant 80$, were not constant values.

Coefficients (a, c_u, c_s) and exponents (b, d_u, d_s), according to the eqns. (5) to (8), depend upon: U_o, U, U_{d_1}, U_{d_n}, R_l, t (s) and the number of steps, n. The mathematical analyses of this simulation will be discussed in a future paper.

TABLE 4

Alkaline manganese LR 20-VARTA
Experimental U_d (V) vs. t_e (s) and simulated time: t_{es}, t_{ss} (s)
Short-period discharge, $n=1...21$
Half of the discharge: $U_d=3U_o/4$, $n=40$
Discharge time, $U_d=U_o/2$, $n=80$

	Sample 3				Sample 4			
	$R_l=10$ Ω, $U=1.5725$ V				$R_l=3.33$ Ω, $U=1.5518$ V			
n	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)
0	1.5910	0	0	0	1.5901	0	0	0
1	1.5676	1	1.00	1.00	1.5317	1	1.00	1.00
2	1.5654	3	3.00	3.00	1.5207	2	2.00	2.00
3	1.5640	4	4.75	4.75	1.5162	4	2.46	2.46
4	1.5630	5	5.40	5.40	1.5130	5	4.69	4.69
5	1.5621	6	6.36	6.36	1.5105	6	5.69	5.69
6	1.5614	7	7.12	7.12	1.5085	7	6.66	6.66
7	1.5607	8	8.17	8.17	1.5068	8	7.64	7.64
8	1.5602	10	8.88	8.88	1.5053	10	8.63	8.63
9	1.5596	11	11.16	11.16	1.5039	11	10.70	10.70
10	1.5592	12	11.80	11.80	1.5027	12	11.65	11.65
11	1.5587	13	13.05	13.05	1.5016	13	12.65	12.65
12	1.5583	14	13.87	13.87	1.5006	14	13.64	13.64
13	1.5579	16	14.91	14.91	1.4997	15	14.62	14.62
14	1.5576	17	16.74	16.74	1.4988	17	15.66	15.66
15	1.5572	18	18.02	18.02	1.4980	18	17.65	17.65
16	1.5569	19	18.79	18.79	1.4972	19	18.69	18.69
17	1.5566	20	19.82	19.82	1.4965	20	19.63	19.63
18	1.5563	21	20.85	20.85	1.4958	21	20.67	20.67
19	1.5560	23	21.88	21.88	1.4952	23	21.60	21.60
20	1.5557	24	23.93	23.93	1.4945	24	23.75	23.75
21	1.4987	2160	2120.08	2120.08	1.4425	360	368.00	368.00
22	1.4733	4320	4365.08	4365.08	1.4195	720	715.00	715.00
23	1.4538	6481	6415	6415	1.4035	1080	1085	1069
40	1.1933	162367	158939	132799	1.1926	21400	22045	20192
80	0.7955	308400	302213	321627	0.7951	92000	94005	93334

Conclusions

This article gives a general overview of the simulation cell/battery discharge curve by short-time discharge method.

The simulation of the cell/battery discharge curve by short-time discharge method (<1% total discharge time) is a good predictor of the cell/battery in long-term performance and reliability.

The usefulness of the simulation depends on:

(i) the time required to close the discharge circuit;
(ii) the precision of the time/voltage data acquisition;
(iii) the elapsed time by the short-time discharge, and

TABLE 5

Alkaline manganese LR 20-VARTA
Experimental U_d (V) vs. t_e (s) and simulated time: t_{es} (s), t_{ss} (s)
Short-period discharge, $n=1...21$
Half of the discharge: $U_d=3U_o/4$, $n=40$
Discharge time, $U_d=U_o/2$, $n=80$

	Sample 5				Sample 6			
	$R_l=3.33\ \Omega$, $U=1.5266$ V				$R_l=3.33\ \Omega$, $U=1.5447$ V			
n	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)	U_d (V)	t_e (s)	t_{se} (s)	t_{ss} (s)
0	1.5895	0	0	0	1.5909	0	0	0
1	1.5120	1	1	1	1.5277	1	1	1
2	1.5040	2	2	2	1.5236	2	2	2
3	1.4998	4	2.59	2.59	1.5178	4	3.76	3.76
4	1.4968	5	4.84	4.84	1.5151	5	5.24	5.24
5	1.4944	6	5.83	5.83	1.5128	6	6.19	6.19
6	1.4924	7	6.81	6.81	1.5110	7	7.03	7.03
7	1.4907	8	7.77	7.77	1.5093	8	8.05	8.05
8	1.4893	10	8.70	8.70	1.5079	10	8.93	8.93
9	1.4880	11	10.75	10.75	1.5066	11	10.98	10.98
10	1.4868	12	11.75	11.75	1.5055	12	11.88	11.88
11	1.4858	13	12.67	12.67	1.5044	13	12.93	12.93
12	1.4848	14	13.72	13.72	1.5035	14	13.80	13.80
13	1.4839	15	14.68	14.68	1.5026	15	14.84	14.84
14	1.4831	17	15.64	15.64	1.5017	17	15.88	15.88
15	1.4823	18	17.70	17.70	1.5010	18	17.74	17.74
16	1.4816	19	18.65	18.65	1.5002	19	18.88	18.88
17	1.4809	20	19.68	19.68	1.4995	20	19.80	19.80
18	1.4803	21	20.61	20.61	1.4989	21	20.72	20.72
19	1.4797	23	21.63	21.63	1.4982	23	21.87	21.87
20	1.4791	24	23.68	23.68	1.4976	24	23.79	23.79
21	1.4325	360	372.00	372.00	1.4472	360	362.00	362.00
22	1.4103	720	712.00	712.00	1.4244	720	696.00	696.00
23	1.3945	1080	1087	1070	1.4083	1080	1075	1058
40	1.1921	19441	19528	18126	1.1932	21961	22216	22016
80	0.7948	87600	85980	87418	0.7955	87000	82294	91813

(iv) the accurateness of the mathematical model representing the power of cell/battery internal resistance.

The cell/battery discharge curve can be considered to be the electrical characteristic ties of a black box.

This method as a fast, nondischarging, nondestructive and repetitive technique is interesting for further investigations.

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List of symbols

U_o	open-circuit voltage, V
U	estimating voltage, V
U_s	simulating voltage, V
U_d	discharge voltage, V
R_i	internal resistance, Ω
C_e	experimental capacity, A s
C_s	simulated capacity, A s
R_l	discharge load, Ω
t_e	experimental time, s
t_s	simulated time, s
P_i	power of R_i, V A
P_d	potential drop, V
a, c_u, c_s	coefficients
b, d_u, d_s	exponents

Subscripts

e	experimental
k, n	ordinal numbers
std	short-time discharge
s	simulated
ttl	total discharge

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